

11.6 Multiple Continuous Random Variables

	Discrete	Continuous
$P[X \in B]$	$\sum_{x \in B} p_X(x)$	$\int_B f_X(x) dx$
$P[(X, Y) \in R]$	$\sum_{(x,y):(x,y) \in R} p_{X,Y}(x, y)$	$\iint_{\{(x,y):(x,y) \in R\}} f_{X,Y}(x, y) dx dy$
Joint to Marginal: (Law of Total Prob.)	$p_X(x) = \sum_y p_{X,Y}(x, y)$ $p_Y(y) = \sum_x p_{X,Y}(x, y)$	$f_X(x) = \int_{-\infty}^{+\infty} f_{X,Y}(x, y) dy$ $f_Y(y) = \int_{-\infty}^{+\infty} f_{X,Y}(x, y) dx$
$P[X > Y]$	$\sum_x \sum_{y: y < x} p_{X,Y}(x, y)$ $= \sum_y \sum_{x: x > y} p_{X,Y}(x, y)$	$\int_{-\infty}^{+\infty} \int_{-\infty}^x f_{X,Y}(x, y) dy dx$ $= \int_{-\infty}^{+\infty} \int_y^{+\infty} f_{X,Y}(x, y) dx dy$
$P[X = Y]$	$\sum_x p_{X,Y}(x, x)$	0
$X \perp\!\!\!\perp Y$	$p_{X,Y}(x, y) = p_X(x)p_Y(y)$	$f_{X,Y}(x, y) = f_X(x)f_Y(y)$
Conditional	$p_{X Y}(x y) = \frac{p_{X,Y}(x,y)}{p_Y(y)}$	$f_{X Y}(x y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$
$\mathbb{E}[X^2 + Y^2]$ $\mathbb{E}[g(X, Y)]$	$\sum_x \sum_y g(x, y) p_{X,Y}(x, y)$	$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(x, y) f_{X,Y}(x, y) dx dy$
$P[g(X, Y) \in B]$	$\sum_{(x,y): g(x,y) \in B} p_{X,Y}(x, y)$	$\iint_{\{(x,y): g(x,y) \in B\}} f_{X,Y}(x, y) dx dy$
$Z = X + Y$	$p_Z(z) = \sum_x p_{X,Y}(x, z - x)$ $= \sum_y p_{X,Y}(z - y, y)$	$f_Z(z) = \int_{-\infty}^{+\infty} f_{X,Y}(x, z - x) dx$ $= \int_{-\infty}^{+\infty} f_{X,Y}(z - y, y) dy$

Table 7: pmf vs. pdf